

Generic Misalignment Aberration Patterns and the Subspace of Benign Misalignment

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ABSTRACT

Q1: Why deploy N wavefront sensors on a three mirror anastigmat (TMA) and not $N + 1$?

Q2: Why measure M Zernike coefficients and not $M + 1$?

Q3: Why control L rigid body degrees of freedom (total) on the secondary and tertiary and not $L + 1$?

The usual answer: “We did a lot of ray tracing and N, M , and L seemed OK.”

We show how straightforward results from aberration theory may be used to address these questions. We consider, in particular, the case of a three mirror anastigmat.

Keywords: Telescopes: wide-field, alignment, wavefront sensing

1. INTRODUCTION

1.1 New Wide Field Telescopes and Wavefront Sensing

The next decade will see the construction of several billion-dollar-class wide field telescopes, ground-based and in space, for which the measurement of cosmological weak gravitational lensing is a major programmatic goal.[†] The image quality delivered by these telescopes must be understood to a level never before achieved by astronomical telescopes.

At least four different effects will contribute to the observed point spread function (henceforth the PSF): telescope tracking errors, optical misalignments, mirror figure errors, and diffusive and transfer effects within the detector. For ground-based telescopes, atmospheric seeing makes an additional contribution.

Wavefront sensors may be deployed to disentangle these effects, and in particular, to measure mirror figure errors and the misalignments of the telescope optics. A recent study of the design for the Large Synoptic Survey Telescope (henceforth the LSST) describes four wavefront sensors measuring Zernike polynomials up to 36th order.¹ But why four, and why 36th order? The answer would appear to be that ray tracing was carried out for many combinations, and that these choices sufficed.

While ray tracing simulations may identify wavefront sensing configurations that are effective and efficient, one might still want to understand *why* they are effective and efficient. And insofar as ray tracing simulations are only approximate in their modeling of telescopes and detectors, an understanding of what lies behind the answers can be quite useful. The purpose of this paper is to present a coherent and concise development of such an understanding.

1.2 Outline of Paper

In §2 we describe the generic 3rd order Seidel misalignment aberration patterns that arise in circularly symmetric telescopes. In §3 we consider the particular case of a three mirror anastigmat, and show that the 3rd order misalignment aberration patterns do not suffice to produce perfect telescope alignment. In §4 we describe the generic 5th order Seidel misalignment aberration patterns that arise in circularly symmetric telescopes. In §5 we introduce the *subspace of benign misalignment* defined by the subset of misalignments that produce none of the third order asymmetric aberration patterns described in §3. In §6 we answer, in part, the questions of the number of wavefront sensors needed and the order to which the wavefront must be measured.

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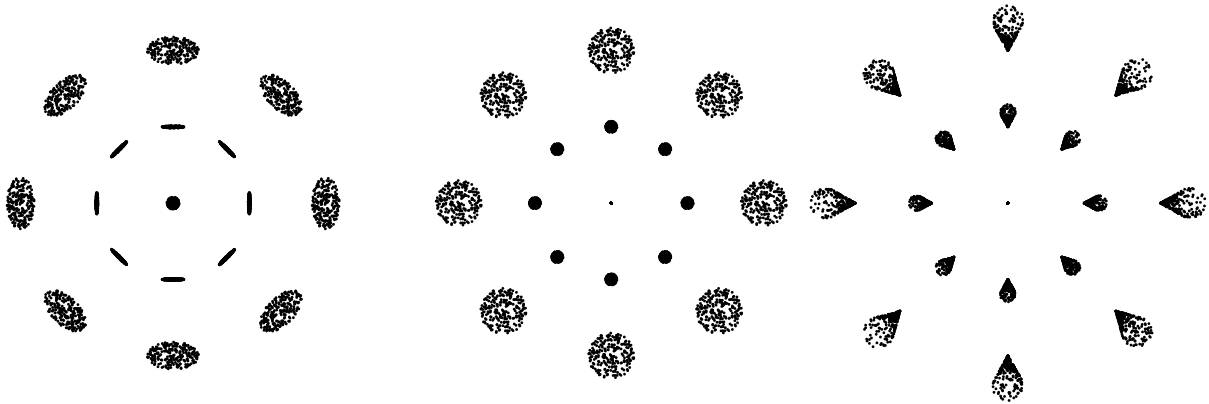


Figure 1. Symmetric 3rd order aberration patterns: astigmatism, curvature of field and coma. Spherical aberration (not shown) is constant across the field. A constant defocus has been added to the astigmatism to show the orientations of the astigmatic images.

2. GENERIC MISALIGNMENT ABERRATION PATTERNS: 3RD ORDER

The development outlined here draws heavily on that of Tessieres,² which in turn draws heavily on that of Thompson and collaborators.³⁻⁵ It is described in considerably more detail by Schechter and Levinson.⁶

2.1 Symmetric Third Order Aberration Patterns

A parabolic one mirror telescope produces four different symmetric third order Seidel aberration patterns at its prime focus: coma, astigmatism, curvature of field and distortion. A Ritchey-Chretien two-mirror telescope is designed to eliminate the symmetric coma pattern at the focus of its secondary. A three mirror anastigmat is designed to remove both the symmetric coma and symmetric astigmatism patterns at the focus of its tertiary.[‡]

Though one may have designed a telescope to eliminate these patterns, they reappear if one has despace errors. Alternatively, if the design hasn't eliminated one of these patterns, despace errors will change its amplitude.

In figure 1 we show the three symmetric third order Seidel aberration patterns that produce imperfect images. Note that some defocus has been added to the astigmatism to show the orientation of the pattern. Each symmetric pattern is described by one number, an amplitude.

2.2 Asymmetric Third Order Aberration Patterns

Decenter and tilt errors produce asymmetric third order Seidel aberration patterns. In contrast with the symmetric patterns, each asymmetric pattern is described by *two* numbers, an amplitude and a direction. These patterns are shown in figure 2. In general a misaligned mirror will produce a superposition of these patterns.

3. THE CASE OF THE THREE MIRROR ANASTIGMAT

3.1 Degrees of Freedom

We consider the particular case of a circularly symmetric three mirror anastigmat (TMA). We take the position and orientation of the primary to define the telescope pointing. Assuming circular symmetry, there are five rigid body degrees of freedom each for the secondary and tertiary, for a *total of ten*. One must therefore measure ten numbers to keep such a telescope perfectly aligned.

[‡]We shall ignore distortion patterns since these produce only wavefront tilts, displacing the positions of otherwise perfectly focused images. One can imagine using preexisting astrometry and science images to measure distortion patterns, which might obviate the need for measuring one or more of the patterns described below.

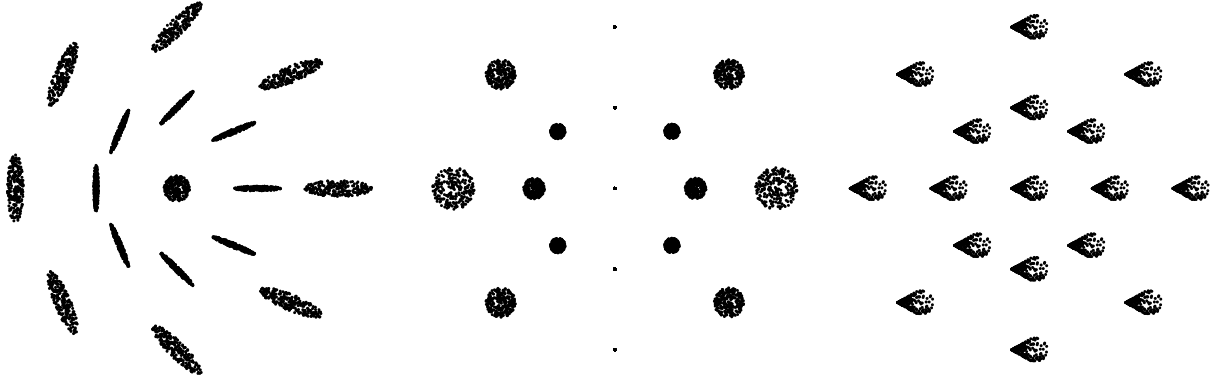


Figure 2. Asymmetric 3rd order aberration patterns: astigmatism, curvature of field and coma.

3.2 Corrected to 3rd Order but Still Imperfectly Aligned

The two symmetric degrees of freedom that must be controlled are the piston of the secondary and the tertiary. There are five aberration patterns that might be used to do this: third order coma, astigmatism, curvature of field and spherical aberration and first order defocus.

This leaves eight asymmetric degrees of freedom. Measuring asymmetric coma, astigmatism and curvature of field (which manifests itself as a tilt of the focal plane) leaves us with two uncorrected asymmetric degrees of freedom. The telescope is still imperfectly aligned.

While one might in principle measure the third order asymmetric distortion patterns, these require a precision astrometric catalog. The alternative is to measure one of the asymmetric fifth order aberration patterns.

4. GENERIC MISALIGNMENT ABERRATION PATTERNS: 5TH ORDER

A three mirror anastigmat eliminates symmetric aberration patterns only to third order. There are two broad classes of symmetric fifth order aberration patterns: coma, astigmatism, defocus and spherical aberration that vary as a higher power of field angle and those that vary more rapidly on the pupil.

4.1 Nomenclature

There is no consistent nomenclature for fifth order Seidel aberrations. We shall refer to the pattern of comatic images that varies as the third power of field angle as *fifth order coma*. Likewise we shall refer to the patterns of astigmatic images, defocussed images, and spherically aberrated images that vary by two powers of field angle more than their third order counterparts as *fifth order astigmatism*, *fifth order defocus* and *fifth order spherical aberration*. These aberrations are all described by low order Zernike polynomials.

In addition to these, there are fifth order aberrations that have no third order counterpart – they produce spot patterns unlike those of third order and are described by higher order Zernike polynomials. An example would be the pattern that varies as $\rho^5 \cos \theta$ on the pupil. We shall refer to this as *coma-II* to distinguish it from conventional coma (coma-I) that varies $\rho^3 \cos \theta$ on the pupil. Likewise we shall refer to the aberration that varies as $\rho^4 \cos 2\theta$ as *astigmatism-II*, and to the aberration that varies as ρ^6 as *spherical-II*.[§]

Rounding out the complement of fifth order Seidel aberrations is *trefoil*, which varies as $\rho^3 \cos 3\theta$ on the pupil.

[§]Were we completely consistent we would use the term *focus-II* in preference to spherical aberration, but this would create rather than eliminate confusion.

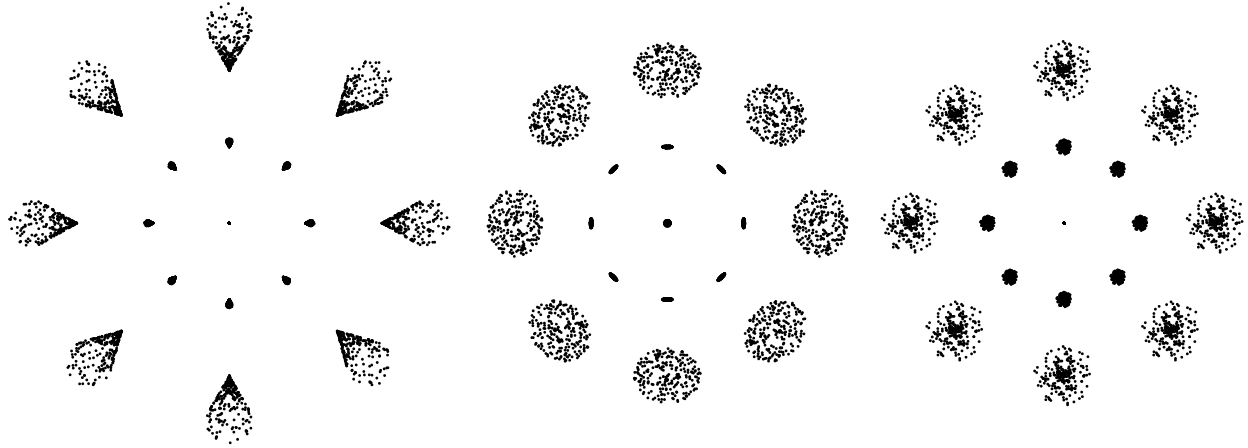


Figure 3. Symmetric 5th order aberration patterns: coma-I, astigmatism-I, and spherical-I. These differ from the patterns in Figure 1 only in the dependence upon field angle and not in the character of the point spread function. Not shown is 5th order defocus.

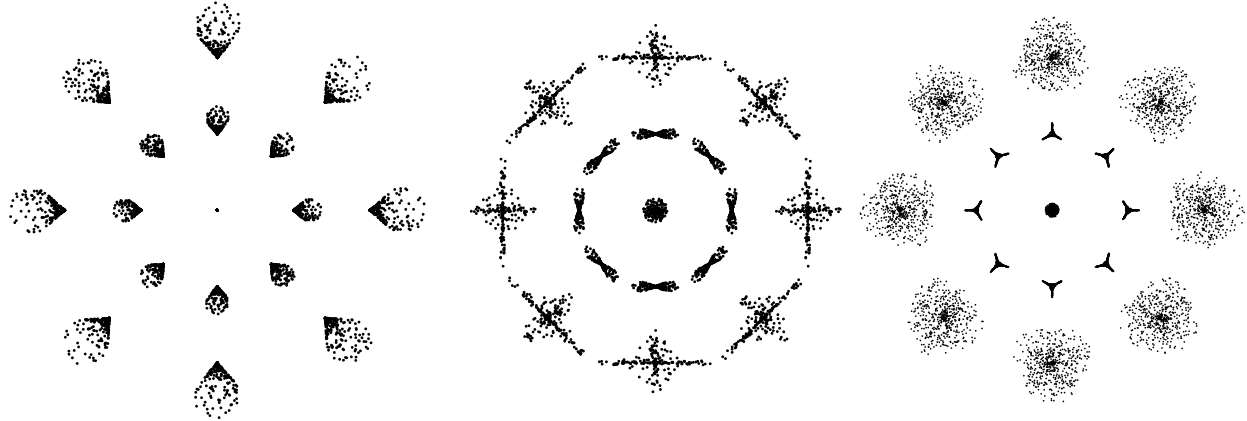


Figure 4. More symmetric 5th order aberration patterns: coma-II, astigmatism-II, and trefoil. Note that the character of the point spread function for coma-II differs from that of the third order coma-I in figure 1, and likewise that for astigmatism-II and astigmatism-I. Not shown is spherical-II, which is constant across the field. A constant spherical aberration has been added to the astigmatism-II to show the orientations of the images. A constant aberration varying as ρ^3 on the pupil has likewise been added to the trefoil.

4.2 Symmetric Fifth Order Aberration Patterns

In addition to the third order aberrations patterns of §2, despace errors produce symmetric fifth order aberration patterns, six of which are shown in Figures 3 and 4. Not shown are spherical-II, which is constant across the field, and fifth order defocus, for which the dependence on field angle is the same as for fifth order astigmatism. A more complete discussion is presented by Schechter and Levinson.⁶

4.3 Asymmetric Fifth Order Aberration Patterns

Tilt and decenter errors also produce asymmetric fifth order aberration patterns. Figure 5 shows *only* those associated with the different types of coma. Asymmetric fifth order coma-II is constant across the field, as was the case for asymmetric third order coma-I. There are two distinct patterns for asymmetric fifth order coma-I.

Asymmetric fifth-order patterns for spherical, trefoil, defocus, astigmatism-II and astigmatism-I can be found in Schechter and Levinson.⁶ As with asymmetric fifth order coma-I, asymmetric fifth order astigmatism-I exhibits two distinct patterns, each of which might be used to constrain two degrees of freedom.

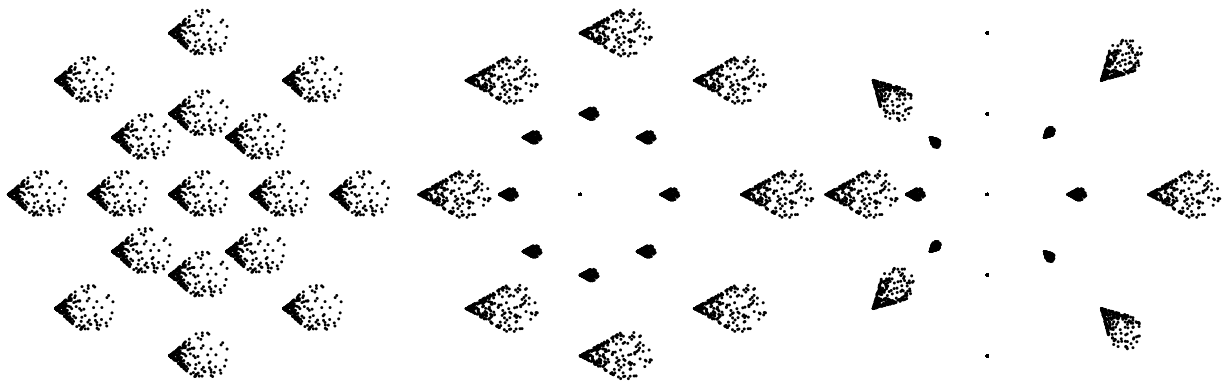


Figure 5. Asymmetric 5th order aberration patterns – The pattern on the left is for coma-II. The pattern in the center and the one on the right are both for coma-I. A similar triplet of patterns might be shown for astigmatism-II and astigmatism-I. There are many more patterns that are not shown.

5. THE SUBSPACE OF BENIGN MISALIGNMENT

Any one of the asymmetric fifth-order patterns presented in the previous section might be used to give the two additional numbers that we have seen are needed to achieve perfect alignment of a TMA. But examination of the *amplitudes* of these patterns shows them to be small, as shown by Schechter and Levinson⁶ and in particular, in Tessieres’² treatment of an early version of the LSST. These small amplitudes make it difficult to achieve perfect alignment.

Conversely, these same small amplitudes imply that one need not strive for perfect alignment. If one eliminates the asymmetric third order aberration patterns, the residual asymmetric fifth order patterns will be small as long as one has not strayed too far from the telescope’s design.

For a TMA there is a two dimensional space of misalignments that produces only such fifth-order asymmetric patterns. We refer to this as the *subspace of benign misalignment* – the subspace in which asymmetric third order coma, astigmatism and curvature of field are all zeroed.

5.1 Ensuring Benign Misalignment

To zero out asymmetric third order coma, astigmatism and curvature of field, one must control six asymmetric degrees of freedom. For our hypothetical TMA this means controlling both the secondary *and* the tertiary.

But until now we have implicitly assumed a perfectly aligned focal plane. Asymmetric curvature of field manifests itself as a tilt of the focal plane. One might therefore, alternatively, control four degrees of freedom on the secondary *and* the tilt of the focal plane.

6. HOW MANY WAVEFRONT SENSORS AND HOW MANY ZERNIKE MODES?

The most straightforward way to keep a TMA aligned would be to measure symmetric first order defocus and third order spherical to control the despace errors and to measure asymmetric coma, astigmatism and curvature of field patterns to stay within the subspace of benign misalignment. The Zernike modes are then limited to the first eight.

Simultaneous measurement of first order defocus and asymmetric curvature of field (which manifests itself as focal plane tilt) requires *a minimum of three* wavefront sensors. One might alternatively use symmetric third order coma (or astigmatism) instead of first order defocus, but the best way to control first order defocus (which is large) is to measure it directly rather than infer it indirectly.

One would also want to measure at least one of the asymmetric fifth order aberration patterns to be sure that one hasn’t travelled too far from the telescope design within the subspace of benign misalignment. The

choice of which pattern to measure will depend upon the expected amplitudes of the patterns and the positions chosen for the wavefront sensors.

Note, by way of example, that three wavefront sensors at the same distance from the center of the field cannot discriminate between asymmetric third-order coma and one of the two kinds of asymmetric fifth-order coma-I, both of which are constant at fixed field angle.

Another consideration is the similarity of certain aberrations. Coma-I and coma-II are quite similar, and will give correlated measurements that in turn, will require high signal to noise to get adequate discrimination.

6.1 Mirror Deformations

While we have concerned ourselves with mirror misalignments, wavefront sensors are likely to be used both for maintaining alignments *and* for maintaining mirror figures. While we have shown that one need only measure low order Zernike modes to maintain alignment, one may need higher order modes to maintain mirror figure.^{7,8}

It is fortunate that none of the misalignment patterns gives constant astigmatism, because the softest bending modes for the primary mirror will typically give such a signal.⁹ But the next softest mode, at least in the case of a primary mirror with a central hole, is likely to be cone-like. Such a deformation, in combination with defocus, looks much like spherical aberration.⁸ One must measure the wavefront to higher Zernike order to separate these.

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